

Kaluza-Klein modes of bulk fields in a generalized Randall-Sundrum scenario

Joydip Mitra * and Soumitra SenGupta †

*Department of Theoretical Physics and Centre for Theoretical Sciences,
Indian Association for the Cultivation of Science,
Kolkata - 700 032, India*

We consider a generalised two brane Randall-Sundrum model with non-zero cosmological constant on the visible TeV brane. Massive Kaluza-Klein modes for various bulk fields namely graviton, gauge field and antisymmetric second rank Kalb-Ramond field in a such generalized Randall-Sundrum scenario are determined. The masses for the Kaluza-Klein excitations of different bulk fields are found to depend on the brane cosmological constant indicating interesting consequences in warped brane particle phenomenology.

I. INTRODUCTION

Theories with extra space-time dimensions are being studied with renewed interest ever-since it was shown that they could provide a solution [1, 2] to the gauge hierarchy problem in connection with the mass of Higgs in the standard model of elementary particles. In the warped geometry model proposed by Randall and Sundrum (RS) [3], one considers a single extra dimension compactified on a S^1/Z_2 orbifold with two flat 3-branes sitting at the two orbifold fixed points. The mass scales in the two branes are hierarchically warped from Planck scale to TeV scale. The brane corresponding to TeV scale is identified as the standard model brane. The brane separation parameter (i.e the modulus) of such a model can be stabilized by incorporating a scalar field in the bulk with suitable potential[4]. For the localization of the standard model fermion fields on the TeV scale 3-brane, two approaches in general are adopted. In one view point the localization is achieved by a bulk scalar field with an appropriate coupling. Alternatively from a string theory angle, the standard model fermions being open string modes are naturally attached and hence localized to the brane. A closed string mode like graviton however can propagate inside the bulk.

In general any bulk field has various Kaluza-Klein (KK) mode excitations which are expected to couple to the brane fields leading to interesting phenomenology. The roles of such KK modes of different bulk fields on the TeV brane have been explored in different works [5, 6, 7, 8, 9, 10] to determine their signatures in the particle phenomenology on the standard model brane. Such phenomenological signatures of course crucially depend on the masses of the Kaluza-Klein modes of these bulk fields. It is seen that in the standard RS scenario, the masses of the KK modes of various bulk fields are suppressed by the warp factor $e^{-kr_c\pi}$ (where r_c is the radius of compactification along the 5-th dimension, k is related to the bulk cosmological constant) so that the low lying KK modes would be characterized by a scale of the order of TeV which naturally may give rise to interesting phenomenology at the TeV scale experiments. Among various possible fields, scalar, graviton, gauge field and Kalb-Ramond field have drawn special attention because of their possible presence in the bulk in string-based models.[11, 12]. The phenomenological implications of the KK mode corresponding to these bulk fields have been studied extensively in the backdrop of Randall-Sundrum model.[13, 14, 15, 16, 17, 18]

Meanwhile there has been an important generalization of the Randall-Sundrum model. In the original model proposed by Randall and Sundrum the TeV-scale 3-brane was chosen to be flat i.e with zero cosmological constant. Subsequently this formalism was generalised to Ricci flat [19] as well as de-Sitter and anti de-sitter 3-branes[20, 21, 22]. It was shown that because of the presence of non-zero cosmological constant on the 3-brane, the warp factor gets modified and one can have different choices for the value of kr for different values of the cosmological constants such that the desired Planck to TeV scale warping can be achieved. Fermion localization as well as the modulus stabilization of this model have been shown in subsequent works [23, 24]

In this work we carefully examine the effect of non-zero cosmological constant (positive or negative) on the KK modes of the bulk fields which in turn will establish the connection between cosmological constants of our universe at various epoch and the scenarios where KK modes play an important role [25, 26]. We begin with a brief review of the generalised RS model in the next section. In the subsequent sections we find out the modifications of the KK mode masses of various bulk fields due to the non-zero cosmological constants on the TeV scale 3-brane.

* Electronic address: tpjm@iacs.res.in

† Electronic address : tpssg@iacs.res.in

II. MODEL

- Randall-Sundrum warped braneworld model [3] has the following features :
- 1) The bulk space-time is anti-de-Sitter (a negative cosmological constant), the effective cosmological constant induced on the TeV/visible brane is zero.
 - 2) The brane tension of the standard model/visible brane is negative.
 - 3) Without introducing any new scale, other than the Planck scale, one can choose the brane separation modulus r_c to have a value M_P^{-1} such that the desired warping can be obtained between the two branes from Planck scale to TeV scale. This immediately resolves the fine tuning / gauge hierarchy problem in connection with the Higgs mass in the standard model.
 - 4) The modulus can be stabilised to the above chosen value by introducing scalar in the bulk [4] without any further fine tuning.

Question arises that can one generalise such a model with a non-zero cosmological constant on the TeV brane with the possibility of rendering it with a positive tension without disturbing the main focus of the work namely the resolution of the gauge hierarchy issue. This was motivated by the facts that the zero cosmological constant of the visible 3-brane is not consistent with the observed small value of the cosmological constant of our Universe and negative tension branes are intrinsically unstable. Such a generalisation was indeed achieved [20] and it has been demonstrated that one can have a more general warp factor which includes branes with non-zero cosmological constant [27] and in certain cases with positive tension for both the branes. We briefly outline the generalised RS model below.

III. GENERALIZED RS MODEL

The warp factor in such a model is obtained by extremising the action,

$$S = \int d^5x \sqrt{-G} (M^3 \mathcal{R} - \Lambda) + \int d^4x \sqrt{-g_i} \mathcal{V}_i \quad (3.1)$$

where Λ is the bulk cosmological constant, \mathcal{R} is the bulk (5-dimensional) Ricci scalar and \mathcal{V}_i is the tension of the i^{th} brane ($i = hid(vis)$ for the hidden (visible) brane). It is shown that a warped geometry results from a constant curvature brane space-time, as opposed to a flat 3-brane space-time. The generalized ansatz for the warped metric is given by,

$$ds^2 = e^{-2A(y)} g_{\mu\nu} dx^\mu dx^\nu + r^2 dy^2 \quad (3.2)$$

where r corresponds to the modulus associated with the extra dimension and μ, ν stands for brane coordinate indices. As in the original RS model, the scalar mass warping is achieved through the warp factor $e^{-A(kr\pi)} = \frac{m}{m_0} = 10^{-n}$ where r is the compact modulus, $k = \sqrt{-\frac{\Lambda}{12M^3}} \sim \text{Planck Mass}$ with the bulk cosmological constant Λ is chosen to be negative. 'n' the warp factor index must be set to 16 to achieve the desired warping and the magnitude of the induced cosmological constant on the brane in this case is non-vanishing in general and is given by $=10^{-N}$ (in Planck units). For the induced brane cosmological constant, $\Omega > 0$ and $\Omega < 0$, the brane metric $g_{\mu\nu}$ may corresponds to some de-sitter or anti de-Sitter space-time for example dS-Schwarzschild and AdS-Schwarzschild space-times respectively [28].

A. Induced brane cosmological constant $\Omega < 0$

For AdS bulk i.e. $\Lambda < 0$, considering the regime for which the induced cosmological constant Ω on the visible brane is negative if one redefines $\omega^2 \equiv -\Omega/3k^2 \geq 0$, then the following solution for the warp factor is obtained :

$$e^{-A} = \omega \cosh \left(\ln \frac{\omega}{c_1} + ky \right) \quad (3.3)$$

where $c_1 = 1 + \sqrt{1 - \omega^2}$ for the warp factor normalized to unity at $y = 0$. One can show that real solution for the warp factor exists if and only if $\omega^2 \leq 10^{-2n}$. This leads to an upper bound for the magnitude of the cosmological constant as $N_{min} = 2n$. So, for $n = 16$, ω^2 is found to be 10^{-32} . For $N = N_{min}$, we get a degenerate solution $x = n \ln 10 + \ln 2$, where $x = kr\pi$. For $N - 2n \gg 1$, the solutions obtained in this case, are

$$x_1 = n \ln 10 + \frac{1}{4} 10^{-(N-2n)}, x_2 = (N - n) \ln 10 + \ln 4 \quad (3.4)$$

Thus, to have the required Planck to TeV scale hierarchy (i.e $n = 16$) one obtains in general two values of x which correspond to two different values of the brane separation modulus r . For example if we take $n = 16$ and $N = 124$,

$$k\pi r_1 \simeq 36.84 + 10^{-93} , \quad k\pi r_2 = 250.07 \quad (3.5)$$

RS value is recovered for $x = n \ln 10$ and $N = \infty$. Moreover at $x = n \ln 10 + \ln 2 = x_0$ (say) and $N = 2n$, ω^2 reaches it's maximum value. Beyond this the magnitude of ω^2 starts to decrease again. One can also obtain the tension of the visible brane for the above two solutions. When $N = N_{min} = 2n$ i.e $x = x_0$ the visible brane tension is zero. For the entire region for which x is less than x_0 the visible brane tension is negative while for x greater than x_0 the visible brane tension is positive. .

B. Induced brane cosmological constant $\Omega > 0$

In this case, the warp factor is given by,

$$e^{-A} = \omega \sinh \left(\ln \frac{c_2}{\omega} - ky \right) , \quad (3.6)$$

where $\omega^2 \equiv \Omega/3k^2$ and $c_2 = 1 + \sqrt{1 + \omega^2}$. In this case there is no bound on the value of ω^2 , and the (positive) cosmological constant on the visible brane can be of arbitrary magnitude. Also, there is a single solution of $kr\pi$ whose precise value will depend on ω^2 and n . The brane tension is negative for the entire range of values of the positive cosmological constant. In Fig.(1) it has been shown how ω^2 is related to the modulus kr . In this plot we have chosen $n = 16$ so that the hierarchy problem can be solved. It is seen that , in both cases, ($\Omega > 0$ and $\Omega < 0$)

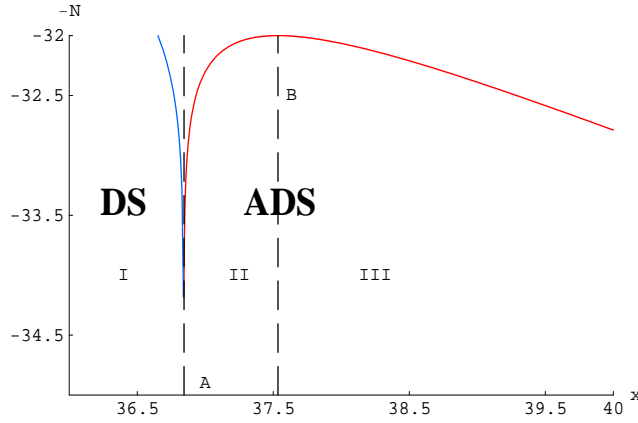


FIG. 1: Graph of N versus $x = kr\pi = 36 - 40$, for $n = 16$ and for both positive and negative brane cosmological constant. The curve in region-I corresponds to positive cosmological constant on the brane, whereas the curve in regions-II & III represents negative cosmological constant on the brane.

the warp factor depends on brane cosmological constant [20] So we expect that KK modes for different bulk fields in this scenario will depend on the brane cosmological constant (Ω).

In this letter, we calculate KK modes and their masses for bulk- gauge field, graviton field and the second rank 2-form anti-symmetric Kalb-Ramond field.

IV. KK MODE FOR FOR DIFFERENT BULK FIELDS FOR ANTI-DE SITTER BRANE ($\Omega < 0$)

In this case, the warp factor is given by,

$$e^{-A} = \omega \cosh \left(\ln \frac{\omega}{c_1} + ky \right) \quad (4.1)$$

where $c_1 = 1 + \sqrt{1 - \omega^2}$.

A. KK Modes for bulk gauge field

Consider bulk $U(1)$ gauge field A_M (where the index M runs over 5 dimensions). Its components A_μ (where μ runs over four dimensions) is Z_2 even and A_4 is Z_2 odd with respect to extra dimension y . A_4 therefore does not have any zero mode in the four-dimensional theory [6, 29]). To start with, we have five-dimensional action S_A for a $U(1)$ gauge theory,

$$S_A = -\frac{1}{4} \int \sqrt{-G} F^{MN} F_{MN} \quad (4.2)$$

where F_{MN} is the five-dimensional field strength tensor given by

$$F_{MN} = \partial_M A_N - \partial_N A_M \quad (4.3)$$

and G is the determinant of the five-dimensional metric. Now, choosing $A_4 = 0$, we decompose A_μ into its KK mode as,

$$A_\mu(x, y) = \sum_{n=0}^{\infty} A_\mu^n(x) \chi_n(y) \quad (4.4)$$

Integrating the eqn (4.2) by parts the action becomes,

$$S_A = \int d^4x \sum_{n=0}^{\infty} \sqrt{-g} \left[-\frac{1}{4} g^{\mu\kappa} g^{\nu\lambda} F_{\kappa\lambda}^n F_{\mu\nu}^n - \frac{1}{2} m_n^2 A_\lambda^n A_\nu^n \right] \quad (4.5)$$

where

$$F_{\mu\nu}^n = \partial_\mu A_\nu^n - \partial_\nu A_\mu^n \quad (4.6)$$

Here g is the determinant of 4-dimensional space-time.

In order to obtain the expression (4.5) we require that the y -dependent wave-function should satisfy the orthonormality condition,

$$\int_0^\pi dy \chi^m(y) \chi^n(y) = \delta^{mn} \quad (4.7)$$

along with the differential equation,

$$-\frac{d}{dy} (e^{-2A(y)} \frac{d\chi^n}{dy}) = m_n^2 \chi^n \quad (4.8)$$

The expression in eq.(4.5) describes the 4-dimensional effective action for the gauge fields A_μ with KK mode masses m_n .

Transforming the variable $z_n = \frac{m_n}{k} e^A$ we get ,

$$\frac{d^2 \chi^n}{dy^2} = k^2 \tanh^2(\ln(\omega/c_1) + ky) \left[z_n^2 \frac{d^2 \chi^n}{dz_n^2} + z_n \frac{d\chi^n}{dz_n} - z_n \operatorname{cosech}^2(\ln(\omega/c_1) + ky) \frac{d\chi^n}{dz_n} \right] \quad (4.9)$$

Now for small ω^2 , the third term in the right hand side of the above equation is negligibly small with respect to the second term and the resulting differential equation becomes,

$$\left[z_n^2 \frac{d^2}{dz_n^2} - z_n \frac{d}{dz_n} + z_n^2 \coth^2(\ln(\omega/c_1) + ky) \right] \chi^n = 0 \quad (4.10)$$

Again, redefining the variable $f = e^{-A(y)} \chi(z)$, the above differential eqn. takes the form,

$$\left[z_n^2 \frac{d^2}{dz_n^2} + z_n \frac{d}{dz_n} + (z_n^2 - 1 + \frac{k^2 \omega^2}{m_n^2} z_n^4) \right] f^n = 0 \quad (4.11)$$

where we have neglected terms proportional to ω^4 . Since, the term $\frac{k^2\omega^2}{m_n^2}z_n^4$ is small compared to $(z_n^2 - 1)$, we can treat this term as a small perturbation. The solution of above differential equation turns out to be,

$$f^n = \frac{1}{N_n} [J_1(z_n) + \alpha_n Y_1(z_n) + \delta(z_n)] \quad (4.12)$$

Here, in absence of the last term, the equation as expected is a first order Bessel equation, [6] and the $\delta(z_n)$ is the contribution due to the perturbation. This term has ω^2 dependence so that in the limit $\omega^2 \rightarrow 0$, we get back the flat space unperturbed solution [6]. $\frac{1}{N_n}$ is Because of the smallness of the perturbation term the normalization constant is taken to be same as in [6]. The solution of eqn.(4.11) therefore can be written as

$$\chi^n(y) = e^A \frac{1}{N_n} [J_1(z_n) + \alpha_n Y_1(z_n) + \delta(z_n)] \quad (4.13)$$

where as stated above, $J_1(z_n)$ and $Y_1(z_n)$ are first order Bessel and Neumann functions and α_n are constant coefficients. It may be recalled that for the unperturbed case, the solution is

$$\chi^n(y) = e^A \frac{1}{N_n} [J_1(z_n) + \alpha_n Y_1(z_n)] \quad (4.14)$$

Hermiticity of the differential operator in above eqn. requires that χ_n and it's first derivative should be continuous at the orbifolded fixed points viz. $y = 0$ and $y = \pi$. This leads to,

$$\alpha_n = -\frac{\pi}{2[\ln(\frac{x_n}{2} - A_\pi + \gamma + \frac{1}{2})]} \quad (4.15)$$

where $x_n = \frac{m_n}{k} e^{A_\pi}$ and A_π is the value of the warp factor at $y = \pi$ which is 10^{-16} . Here because of the correction to α_n beyond ω^2 has been neglected.

Now, we have the differential eqn. for $\delta(z_n)$ as (substituting f^n in eqn.(4.11))

$$z_n^2 \frac{d^2 \delta}{dz_n^2} + z_n \frac{d\delta}{dz_n} + (z_n^2 - 1)\delta + (\frac{k^2\omega^2}{m_n^2} z_n^4)(J_1(z_n) + \alpha_n Y_1(z_n)) = 0 \quad (4.16)$$

Taking the leading order terms in $J_1(z_n)$ and $\alpha_n Y_1(z_n)$, the equation turns out to be

$$\left[z_n^2 \frac{d^2}{dz_n^2} + z_n \frac{d}{dz_n} + (z_n^2 - 1) \right] \delta(z_n) + \frac{k^2\omega^2}{m_n^2} z_n^5 = 0 \quad (4.17)$$

The solution of the above equation can be written as,

$$\begin{aligned} \delta(z_n) = & \frac{1}{2} [(-4a^2 \pi z_n^3 J_3(z_n) Y_1(z_n) + a^2 \pi z_n^4 J_4(z_n) Y_1(z_n)) \\ & + 16a^2 \pi J_1(z_n) MeigerG[\{\{1\}, \{\frac{3}{2}\}\}, \{\{2, 3\}, \{0, \frac{3}{2}\}\}, z_n/2, 1/2]] \end{aligned} \quad (4.18)$$

The perturbed solution is therefore,

$$\chi^n(y) = \frac{e^A [J_1(z_n) + \alpha_n Y_1(z_n) + \delta(z_n)]}{N_n} \quad (4.19)$$

Now, we calculate $\frac{d\chi}{dy}$ at $y = \pi$ and put it equal to zero. The roots of this equation yield the modified values of the KK modes in the generalized RS scenario. Calculating the derivative at $y = \pi$, we get

$$x_n^N [J_1'(x_n^N) + \alpha_n Y_1'(x_n^N) + \delta'(x_n^N)] + [J_1(x_n^N) + \alpha_n Y_1(x_n^N) + \delta(x_n^N)] = 0 \quad (4.20)$$

where (x_n^N) denotes the root for the perturbed case. Now, expanding

$$J_1'(x_n^N) = J_1'(x_n^0) + \partial_{x_n} J_1'(x_n)|_{x_n^0} \frac{e^A}{k} \Delta m_n \quad (4.21)$$

we obtain Δm_n which in turn gives the value of the shifted root where x_n^0 are the roots for unperturbed solution [6]. Using the similar Taylor series expansion, for $\delta(x_n^N) = \delta(x_n^0)$ (Keeping only up to the leading order term), we finally arrive at,

$$\Delta m_n = \frac{x_n^0 \delta'(x_n^0) e^{-A_\pi} k + \delta(x_n^0) e^{-A_\pi} k}{x_n^0 J_1''(x_n^0) + x_n^0 \alpha_n Y_1''(x_n^0) + 2J_1'(x_n^0) + 2\alpha_n Y_1'(x_n^0)} \quad (4.22)$$

So, corresponding to the old roots $x_1^0 = 2.45, x_2^0 = 5.57, x_3^0 = 8.70$, we get the new roots such that $\Delta m_1 = 0.0027 \star 10^{35} \omega^2$; $\Delta m_2 = 0.0108 \star 10^{35} \omega^2$; $\Delta m_3 = 0.0144 \star 10^{35} \omega^2$ etc.

Therefore, Δm_n gives the correction to the unperturbed kk mode masses for the bulk gauge field in the generalized RS scenario.

B. KK Modes for the anti-symmetric two form Kalb Ramond field

Here we consider rank-2 antisymmetric tensor field (Kalb-Ramond field) together with gravity in the bulk. It is well known that the third rank tensor field strength corresponding to the Kalb-Ramond field can be identified with space-time torsion[30]. The rank-3 antisymmetric field strength tensor H_{MNL} is related to the KR field B_{MN} [31] as

$$H_{MNL} = \partial_{[M} B_{NL]} \quad (4.23)$$

The 5-dimensional action for the curvature-torsion sector in this case is

$$\mathcal{S}_G = \int d^4x \int dy \sqrt{-G} 2 M^3 R(G, H) \quad (4.24)$$

Now, this action can be decomposed into two independent parts – one consisting of pure curvature and the other, of torsion,

$$\mathcal{S}_G = \int d^4x \int dy \sqrt{-G} 2 [M^3 R(G) - H_{MNL} H^{MNL}] \quad (4.25)$$

with H_{MNL} related to KR field B_{NL} as in (4.23). The 5-dimensional action corresponding to the KR field therefore is given by

$$\mathcal{S}_H = \int d^4x \int dy \sqrt{-G} H_{MNL} H^{MNL} \quad (4.26)$$

As we use the gauge fixing condition in case of bulk gauge field, here also we use KR gauge fixing condition $B_{4\mu} = 0$. We now explicitly use the generalized RS metric to calculate the above action with the KK mode decomposition for the Kalb-Ramond field as,

$$B_{\mu\nu}(x, y) = \sum_{n=0}^{\infty} B_{\mu\nu}^n(x) \chi^n(y) \quad (4.27)$$

In terms of the 4-D projections $B_{\mu\nu}^n$, an effective action of the form

$$\mathcal{S}_H = \int d^4x \sum_{n=0}^{\infty} \sqrt{-g} [g^{\mu\alpha} g^{\nu\beta} g^{\lambda\gamma} H_{\mu\nu\lambda}^n H_{\alpha\beta\gamma}^n + 3m_n^2 g^{\mu\alpha} g^{\nu\beta} B_{\mu\nu}^n B_{\alpha\beta}^n] \quad (4.28)$$

can be obtained provided the following equation is satisfied,

$$-\frac{d^2 \chi^n}{dy^2} = m_n^2 \chi^n e^{2A(y)} \quad (4.29)$$

along with the following orthogonality condition,

$$\int e^{2A(y)} \chi^m(y) \chi^n(y) dy = \delta_{mn} \quad (4.30)$$

Here $H_{\mu\nu\lambda}^n = \partial_{[\mu} B_{\nu\lambda]}^n$ and $\sqrt{3}m_n$ gives the mass of the n th mode. In terms of $z_n = \frac{m_n}{k} e^{A(y)}$, equation (4.29) can be recast in the form

$$\left[z_n^2 \frac{d^2}{dz_n^2} + z_n \frac{d}{dz_n} - \frac{a^2 z_n^3}{(1 - a^2 z_n^2)} \frac{d}{dz_n} + \frac{z_n^2}{1 - a^2 z_n^2} \right] \chi^n = 0 \quad (4.31)$$

where $a^2 = \frac{k^2 \omega^2}{m_n^2}$. Applying the same argument (as is done for the bulk gauge field), the 3rd term is small compared to z_n , hence this term is neglected. Treating the last term perturbatively, we arrive at the following differential equation,

$$\left[z_n^2 \frac{d^2}{dz_n^2} + z_n \frac{d}{dz_n} + z_n^2(1 + a^2 z_n^2) \right] \chi^n = 0 \quad (4.32)$$

The soln. of the above equation can be written as,

$$\chi^n(y) = 1/N_n [J_0(z_n) + \alpha_n Y_0(z_n) + \delta(z_n)] \quad (4.33)$$

The normalization constant can be found out from the orthogonality condition. $\delta(z_n)$ is the perturbation term and it contains ω^2 factor, so that as $\omega^2 \rightarrow 0$ we get back the unperturbed solution [30, 32]. Now, from the continuity conditions at $y = 0$ and $y = \pi$, α_n and m_n can be found out. Using the fact that $e^{-A_\pi} \gg 1$ and $m_n \ll k$, we obtain from the continuity condition at $y = 0$,

$$\alpha_n \simeq x_n e^{-A_\pi} \quad (4.34)$$

with $x_n = z_n(\pi)$. As ω^2 is small, it's dependence on α_n can be neglected (just as in case of bulk gauge field). Again since, $x_n \simeq 1$, α_n can be neglected ($\alpha_n \ll 1$).

We recall that for the unperturbed case, the boundary condition at $y = \pi$ gives,

$$J_1(x_n) \simeq \frac{\pi}{2} x_n e^{-A_\pi} \quad (4.35)$$

As the right-hand side of the above equation is negligibly small, the roots can be approximated to the zeroes of $J_1(x_n)$.

Now, keeping this approach in mind, we consider the perturbed case. We are interested to find how the roots are modified if $J_1(x_n^N) = 0$ where x_n^N are the new roots because of the modified warp factor in the generalized RS case. If we substitute the solution (4.33) in the eqn.(4.32) we get the differential for $\delta(z_n)$ as

$$\left[z_n^2 \frac{d^2}{dz_n^2} + z_n \frac{d}{dz_n} + z_n^2 \right] \delta(z_n) + a^2 z_n^4 J_0(z_n) = 0 \quad (4.36)$$

now, to the leading order, $J_0(z_n) = \frac{1}{2}$ and the above differential equation can be rewritten as,

$$\left[z_n^2 \frac{d^2}{dz_n^2} + z_n \frac{d}{dz_n} + z_n^2 \right] \delta(z_n) + \frac{1}{2} a^2 z_n^4 = 0 \quad (4.37)$$

The solution of the above differential equation is given by,

$$\begin{aligned} \delta(z_n) = & \frac{1}{4} [2a^2 \pi z_n^2 Y_0(z_n) - 4a^2 \pi z_n J_0(z_n) J_1(z_n) Y_0(z_n) - 2a^2 \pi z_n^2 J_2(z_n) Y_0(z_n) \\ & + a^2 \pi z_n^3 J_3(z_n) Y_0(z_n) - 4a^2 \pi z_n J_0(z_n) Y_1(z_n) + a^2 \pi z_n^3 J_0(z_n) Y_1(z_n) + 4a^2 \pi z_n J_0(z_n)^2 Y_1(z_n)] \end{aligned} \quad (4.38)$$

Since, α_n is negligibly small, we take the solution as

$$\chi^n(y) = J_0(z_n) + \delta(z_n) \quad (4.39)$$

Performing the derivative at $y = \pi$, we get

$$\chi'^n(y) = -J'_1(x_n) + \delta'(x_n) \quad (4.40)$$

Requiring the fact that $\chi^n(y)$ is zero, we get

$$J'_1(x_n^N) + \delta'(x_n^N) = 0 \quad (4.41)$$

. Performing the Taylor series expansion w.r.t x_n^0 (as was done in case of the bulk gauge field) we find the mass correction as

$$\Delta m_n = \frac{\delta'(x_n^0) e^{-A_\pi} k}{J_2(x_n^0)} \quad (4.42)$$

$\Delta m_1 = 0.000647 \star 10^{35} \omega^2$; $\Delta m_2 = 0.000475 \star 10^{35} \omega^2$; $\Delta m_3 = 0.000395 \star 10^{35} \omega^2$ corresponding to $m_1 = 3.83, m_2 = 7.015, m_3 = 10.173$ respectively. Here m_1, m_2, m_3 denotes the masses for the unperturbed case. Thus Δm_n gives the correction to the unperturbed kk masses in case of Kalb-Ramond field in the generalized RS scenario.

C. KK modes for the Graviton Field

Here, same approach is taken as before. Parametrization of the tensor fluctuations $h_{\alpha\beta}$ has been done by taking a linear expansion of the flat metric about it's Ricci flat value, $\hat{G}_{\alpha\beta} = e^{-2A(y)}(g_{\alpha\beta} + \kappa^* h_{\alpha\beta})$, where κ^* is an expansion parameter [10, 33] In order to calculate mass spectrum of tensor fluctuations, we consider 4-dimensional $\alpha\beta$ components with the replacement $G_{\alpha\beta} \rightarrow \hat{G}_{\alpha\beta}$, keeping terms up to $\mathcal{O}(\kappa^*)$. We work in the gauge with $h_\alpha^\alpha = 0$. Expanding $h_{\alpha\beta}$ into a KK tower

$$h_{\alpha\beta}(x, y) = \sum_{n=0}^{\infty} h_{\alpha\beta}^n(x) \chi^n(y) \quad (4.43)$$

. we obtain the equation of motion of $h_{\alpha\beta}^n$ as,

$$\sqrt{-g}(g^{\alpha\beta} \partial_\alpha \partial_\beta - m_n^2) h_{\mu\nu}^n(x) = 0 \quad (4.44)$$

In order to obtain the eqn. in the above form, the following two conditions must be satisfied for $\chi^n(y)$

$$-\frac{d}{dy}(e^{-4A(y)} \frac{d\chi^n}{dy}) = m_n^2 e^{-2A(y)} \chi^n \quad (4.45)$$

and

$$\int e^{-2A(y)} \chi^m(y) \chi^n(y) dy = \delta_{mn} \quad (4.46)$$

The latter defines the orthogonality relation for $\chi^m(y)$. Now transforming the variable $z_n = \frac{m_n}{k} e^{A(y)}$ and defining $\chi(z) = e^{2A} f$ we get the differential equation as,

$$\left[z_n^2 \frac{d^2 \delta}{dz_n^2} + z_n \frac{d\delta}{dz_n} + (z_n^2 - 4)\delta + \frac{k^2 \omega^2}{m_n^2} z_n^4 \right] f^n = 0 \quad (4.47)$$

Since, as stated before, $\frac{k^2 \omega^2}{m_n^2}$ is very small, we can write the solution of the above differential equation as

$$f^n = \frac{1}{N_n} [J_2(z_n) + \alpha_n Y_2(z_n) + \delta(z_n)] \quad (4.48)$$

Since for the unperturbed case [10] we had $\alpha_n Y_2(z_n)$ to be very small in the limit $m_n/k \ll 1$ and $e^{A_\pi} \gg 1$, here also we neglect that dependence and write the solution as

$$f^n = \frac{1}{N_n} [J_2(z_n) + \delta(z_n)] \quad (4.49)$$

Putting this solution into the above equation, we obtain the equation as,

$$\left[z_n^2 \frac{d^2}{dz_n^2} + z_n \frac{d}{dz_n} + (z_n^2 - 4) \right] \delta + \frac{1}{8} \frac{k^2 \omega^2}{m_n^2} z_n^6 = 0 \quad (4.50)$$

as $J_2(z_n) = \frac{1}{8} z_n^2$. (to the leading order) The solution of $\delta(z_n)$ is given by,

$$\begin{aligned} \delta(z_n) = & \frac{1}{16} [(-6a^2 \pi z_n^4 J_4(z_n) Y_2(z_n) + a^2 \pi z_n^5 J_5(z_n) Y_2(z_n) \\ & + 32a^2 \pi J_2(z_n) Meigerg[\{\{1\}, \{\frac{3}{2}\}\}, \{\{2, 4\}, \{0, \frac{3}{2}\}\}, z_n/2, 1/2]] \end{aligned} \quad (4.51)$$

The complete solution therefore becomes

$$\chi^n(y) = \frac{e^{2A} [J_2(z_n) + \delta(z_n)]}{N_n} \quad (4.52)$$

Now, just as we have done in the gauge field case, We calculate the derivative of $\chi(y)$ with respect to y at $y = \pi$ and set it to zero. From the shifted root, we get the correction to the different KK modes. Calculating the derivative at $y = \pi$ we get

$$x_n^N [J_2'(x_n^N) + \delta'(x_n^N)] + 2[J_2(x_n^N) + \delta(x_n^N)] = 0 \quad (4.53)$$

where (x_n^N) denotes the new root. Since we can write,

$$J_2'(x_n^N) = J_2'(x_n^0) + \partial_{x_n} J_2'(x_n)|_{x_n^0} \frac{e^A}{k} \Delta m_n \quad (4.54)$$

and

$$x_n^N = x_n^N - x_n^0 + x_n^0 = \Delta m_n \frac{e^A}{k} + x_n^0 \quad (4.55)$$

Where x_n^0 are the roots obtained in the unperturbed case. Treating $\delta(x_n^N) = \delta(x_n^0)$, we arrive at,

$$\Delta m_n = \frac{x_n^0 \delta'(x_n^0) e^{-A\pi} k + 2\delta(x_n^0) e^{-A\pi} k}{x_n^0 J_2''(x_n^0) + 3J_2'(x_n^0)} \quad (4.56)$$

From these we obtain the correction to the mass values $\Delta m_1 = 0.02105 \star 10^{35} \omega^2$; $\Delta m_2 = 0.11751 \star 10^{35} \omega^2$; $\Delta m_3 = .30584 \star 10^{35} \omega^2$ corresponding to $m_1 = 3.83$, $m_2 = 7.015$, $m_3 = 10.173$ respectively which are the roots of $J_1(x_n^0)$.

Different bulk fields	$n = 1$		$n = 2$		$n = 3$	
	$m_n(\text{in TeV})$	$\Delta m_n(\text{in TeV})$	$m_n(\text{in TeV})$	$\Delta m_n(\text{in TeV})$	$m_n(\text{in TeV})$	$\Delta m_n(\text{in TeV})$
Graviton field	3.83	$0.021 \times 10^{35} \omega^2$	7.02	$0.117 \times 10^{35} \omega^2$	10.17	$0.305 \times 10^{35} \omega^2$
Kalb-Ramond	6.63	$0.00064 \times 10^{35} \omega^2$	12.14	$0.00047 \times 10^{35} \omega^2$	17.28	$0.00039 \times 10^{35} \omega^2$
Gauge -field	2.45	$0.0027 \times 10^{35} \omega^2$	5.57	$0.0108 \times 10^{35} \omega^2$	8.7	$0.0144 \times 10^{35} \omega^2$

TABLE I: The masses of a few low-lying modes for different tensorial fields for $kr_c = 12$ and $k = 10^{19} \text{GeV}$ in case of Ads space.

V. KK MODES FOR DIFFERENT BULK FIELDS FOR DE-SITTER BRANE ($\Omega > 0$)

We can calculate the similar mass splittings in this generalized scenario when the induced brane cosmological constant $\Omega > 0$. The warp factor in this case is

$$e^{-A} = \omega \sinh \left(\ln \frac{c_2}{\omega} - ky \right), \quad (5.1)$$

where $c_2 = 1 + \sqrt{1 + \omega^2}$. Due to different warp factor, the differential equation for the perturbed solution $\delta(z_n)$ changes and thereby its solution will change automatically.

A. KK modes for the bulk Gauge field

Here Applying the exact procedure as in case of ADS bulk, we get the differential equation as

$$\left[z_n^2 \frac{d^2}{dz^2} + z_n \frac{d}{dz} + (z_n^2 - 1) \right] \delta - \frac{k^2 \omega^2}{m_n^2} z_n^5 = 0 \quad (5.2)$$

The solution of the above differential equation as,

$$\begin{aligned} \delta(z_n) = & \frac{1}{2} [(4a^2 \pi z_n^3 J_3(z_n) Y_1(z_n) - a^2 \pi z_n^4 J_4(z_n) Y_1(z_n)) \\ & - 16a^2 \pi J_1(z_n) MeigerG[\{\{1\}, \{\frac{3}{2}\}\}, \{\{2, 3\}, \{0, \frac{3}{2}\}\}, z_n/2, 1/2]] \end{aligned} \quad (5.3)$$

The correction to the mass is given by the equation

$$\Delta m_n = \frac{x_n^0 \delta'(x_n^0) e^{-A_\pi k} + \delta(x_n^0) e^{-A_\pi k}}{x_n^0 J_1''(x_n^0) + x_n^0 \alpha_n Y_1''(x_n^0) + 2J_1'(x_n^0) + 2\alpha_n(x_n^0)} \quad (5.4)$$

So, corresponding to the unperturbed roots $x_1^0 = 2.45, x_2^0 = 5.57, x_3^0 = 8.70$, we get the new roots $\Delta m_1 = 0.0027 \star 10^{35} \omega^2; \Delta m_2 = 0.0108 \star 10^{35} \omega^2; \Delta m_3 = 0.0144 \star 10^{35} \omega^2$ etc.

B. KK modes for the Kalb-Ramond field

In this case, The differential equation will be given by

$$\left[z_n^2 \frac{d^2}{dz_n^2} + z_n \frac{d}{dz_n} + z_n^2 \right] \delta(z_n) - a^2 z_n^4 J_0(z_n) = 0 \quad (5.5)$$

So the solution is now given by,

$$\begin{aligned} \delta(z_n) = & \frac{1}{4} [-2a^2 \pi z_n^2 Y_0(z_n) + 4a^2 \pi z_n J_0(z_n) J_1(z_n) Y_0(z_n) + 2a^2 \pi z_n^2 J_2(z_n) Y_0(z_n) \\ & - a^2 \pi z_n^3 J_3(z_n) Y_0(z_n) + 4a^2 \pi z_n J_0(z_n) Y_1(z_n) - a^2 \pi z_n^3 J_0(z_n) Y_1(z_n) - 4a^2 \pi z_n J_0(z_n)^2 Y_1(z_n)] \end{aligned} \quad (5.6)$$

applying the same procedure as before, we get Δm_n as

$$\Delta m_n = \frac{\delta'(x_n^0)}{J_2(x_n^0)} k e^{-A_\pi} \quad (5.7)$$

. So, for same values of x_1^0, x_2^0, x_3^0 (which has been written in case of ADS bulk, we get $\Delta m_1 = 0.000647 \star 10^{35} \omega^2; \Delta m_2 = 0.000475 \star 10^{35} \omega^2; \Delta m_3 = 0.000395 \star 10^{35} \omega^2$

C. KK modes for the Graviton field

The last case that we have studied is the Graviton in the ADS bulk. In this case, the differential equation is given by

$$\left[z_n^2 \frac{d^2}{dz_n^2} + z_n \frac{d}{dz_n} + (z_n^2 - 4) \right] \delta - \frac{1}{8} \frac{k^2 \omega^2}{m_n^2} z_n^6 = 0 \quad (5.8)$$

as $J(z_n) = \frac{1}{8} z_n^2$ (to the leading order), The solution of $\delta(z_n)$ is given by,

$$\begin{aligned} \delta(z_n) = & \frac{1}{16} [(6a^2 \pi z_n^4 J_4(z_n) Y_2(z_n) - a^2 \pi z_n^5 J_5(z_n) Y_2(z_n) \\ & - 32a^2 \pi J_2(z_n) MeierG[\{\{1\}, \{\frac{3}{2}\}\}, \{\{2, 4\}, \{0, \frac{3}{2}\}\}, z_n/2, 1/2]] \end{aligned} \quad (5.9)$$

Δm_n in this case, is given by,

$$\Delta m_n = \frac{x_n^0 \delta'(x_n^0) e^{-A_\pi k} + 2\delta(x_n^0) e^{-A_\pi k}}{x_n^0 J_2''(x_n^0) + 3J_2'(x_n^0)} \quad (5.10)$$

from these we get the correction to the mass values $\Delta m_1 = 0.02105 \star 10^{35} \omega^2; \Delta m_2 = 0.11751 \star 10^{35} \omega^2; \Delta m_3 = 0.30584 \star 10^{35} \omega^2$ corresponding to $m_1 = 3.83, m_2 = 7.015, m_3 = 10.173$ respectively which are basically the roots of $J(x_n^0)$.

Different bulk fields	$n = 1$		$n = 2$		$n = 3$	
	$m_n(\text{in TeV})$	$\Delta m_n(\text{in TeV})$	$m_n(\text{in TeV})$	$\Delta m_n(\text{in TeV})$	$m_n(\text{in TeV})$	$\Delta m_n(\text{in TeV})$
Graviton field	3.83	$0.021 \times 10^{35} \omega^2$	7.02	$0.117 \times 10^{35} \omega^2$	10.17	$0.305 \times 10^{35} \omega^2$
Kalb-Ramond	6.63	$0.0006 \times 10^{35} \omega^2$	12.14	$0.00047 \times 10^{35} \omega^2$	17.28	$0.00039 \times 10^{35} \omega^2$
Gauge -field	2.45	$0.0027 \times 10^{35} \omega^2$	5.57	$0.0108 \times 10^{35} \omega^2$	8.7	$0.0144 \times 10^{35} \omega^2$

TABLE II: The masses of a few low-lying modes for different tensorial fields for $kr_c = 12$ and $k = 10^{19} \text{ GeV}$ in case of ds space.

VI. CONCLUSION

In this work we have derived the modifications of the KK mode masses for the various bulk fields because of a non-zero cosmological constant (ω^2) on the visible brane. We have shown that for both negative and positive brane cosmological constant (i.e for anti de-Sitter and de-Sitter universe) the leading order corrections to the KK mode masses are proportional to ω^2 . It was discussed earlier that while for de-Sitter case there is no bound for the value of the cosmological constant, for anti de-Sitter case the magnitude can be at most $\sim 10^{-32}$ if one wants to resolve the hierarchy problem consistently. Though the present observed value of the cosmological constant is tiny and positive with $\omega^2 \sim 10^{-124}$ (in Planck unit), it may however be argued that an anti de-Sitter universe with a relatively large negative brane cosmological constant say $\sim 10^{-32}$ (inherited from the bulk) may subsequently have evolved into a de-Sitter universe with a tiny positive value of the brane cosmological constant because of other effects on the brane which may induce positive cosmological constant on the brane. Such a scenario will then indicate the possibility of a large correction to the KK modes of the bulk fields resulting into significant modifications (from a flat brane Randall-Sundrum scenario) of experimental signatures of various processes involving the bulk and the standard model fields.

Acknowledgments

JM acknowledges CSIR, Govt. of India for providing financial support. JM would like to thank S. Majhi for useful suggestions.

-
- [1] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. **B 429**, 263 (1998); *ibid* Phys.Rev. **D 59**, 086004 (1999); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. **B436** 257 (1998).K.Akama, Prog. Theor. Phys. 80, 935 (1988)
 - [2] A. G. Cohen and D. B. Kaplan, Phys. Lett. **B470** , 52 (1999); I. Antoniadis, S. Dimopoulos and A. Giveon, JHEP, **05**, 055 (2001); T. Multamaki and I. Vilja, Phys. Lett. **B545**, 389 (2002); C. P. Burgess, J. M. Cline, N. R. Constable and H. Firouzjahi, JHEP, **01**, 014 (2002)
 - [3] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999)
 - [4] W. D. Goldberger and M. B. Wise, Phys. Rev. Lett. **83**, 4922 (1999)
 - [5] W. D. Goldberger and M. B. Wise, Phys. Rev. D **60**, 107505 (1999)
 - [6] H. Davoudiasl, J. L. Hewett and T. G. Rizzo, Phys. Lett. B **473**, 43 (2000)
 - [7] H. Davoudiasl, J. L. Hewett and T. G. Rizzo, Phys. Rev. D **68**, 045002 (2003)
 - [8] H. Davoudiasl, J. L. Hewett and T. G. Rizzo, JHEP **0308**, 034 (2003)
 - [9] K. Agashe, N. G. Deshpande and G. H. Wu, Phys. Lett. B **511**, 85 (2001) [arXiv:hep-ph/0103235]
 - [10] H. Davoudiasl, J. L. Hewett and T. G. Rizzo, Phys. Rev. Lett. **84**, 2080 (2000)
 - [11] *Superstring Theory*, M. B. Green, J. H. Schwarz and E. Witten, Cambridge University Press, Cambridge (1987)
 - [12] O. Lebedev, Phys. Rev. D **65**, 124008 (2002) [arXiv:hep-ph/0201125]
 - [13] H. Davoudiasl, arXiv:0810.0194 [hep-ph]
 - [14] C. Bambi and F. R. Urban, Phys. Rev. D **78**, 103515 (2008) [arXiv:0808.3500 [hep-ph]]
 - [15] B. Grzadkowski and J. F. Gunion, Phys. Lett. B **653**, 307 (2007)
 - [16] T. G. Rizzo, Phys. Lett. B **647**, 43 (2007) [arXiv:hep-ph/0611224]
 - [17] A. L. Fitzpatrick, J. Kaplan, L. Randall and L. T. Wang, JHEP **0709**, 013 (2007)
 - [18] D. Dominici, B. Grzadkowski, J. F. Gunion and M. Toharia, Nucl. Phys. B **671**, 243 (2003) [arXiv:hep-ph/0206192]
 - [19] A. Chamblin, S. W. Hawking and H. S. Reall, Phys.Rev.D **61**,065007 (2000)
 - [20] S. Das, D. Maity and S. Sengupta, JHEP **05**, 042 (2008)
 - [21] C. Schmidhuber, Nucl. Phys. B **580**, 140 (2000) [arXiv:hep-th/9912156]
 - [22] N. Kaloper Phys. Rev. D **60**, 123506 (1999)

- [23] R. Koley, J. Mitra and S. SenGupta, Phys. Rev. D **79**, 041902 (2009) [arXiv:0806.0455 [hep-th]]
- [24] R. Koley, J. Mitra and S. SenGupta, Europhys. Lett. **85**, 41001 (2009) [arXiv:0809.4102 [hep-th]]
- [25] N. Muta and N. Uekusa, arXiv:hep-ph/0503017
- [26] Y. Aghababaie, C. P. Burgess, S. L. Parameswaran and F. Quevedo, Nucl. Phys. B **680**, 389 (2004) [arXiv:hep-th/0304256]
- [27] M. Sasaki, T. Shiromizu and K. i. Maeda, Phys. Rev. D **62**, 024008 (2000)
- [28] A. Karch and L. Randall, JHEP **0105**, 008 (2001)
- [29] Y. Grossman and M. Neubert, Phys. Lett. B **474**, 361 (2000)
- [30] B. Mukhopadhyaya, S. Sen and S. SenGupta, Phys. Rev. Lett. **89**, 121101 (2002) [Erratum-ibid. **89**, 259902 (2002)]
- [31] M. Kalb and P. Ramond, Phys. Rev. D **9**, 2273 (1974)
- [32] B. Mukhopadhyaya, S. Sen and S. SenGupta, Phys. Rev. D **76**, 121501 (2007)
- [33] H. Davoudiasl, J. L. Hewett and T. G. Rizzo, JHEP **0308**, 034 (2003)